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STATISTICAL ANALYSIS OF THE VARIABILITY OF QUASAR 3C 273

by

L. M. Ozernoy and V. E. Chertoprud

ANNOTATION

The statistical results of observation data relative to cyclical changes in brightness[10] of quasar 3C 273 are given herein. On the basis of an analysis of random deviations of signals from the mean statistical trajectory we obtained some curves of a system approximated by the simplest vibrational equation with one degree of freedom (1a). The period of dynamic movement $T = 9$ years (2), and the mean rigidity characterizing the asymptotic stability of the limiting cycle is $\tilde{N} \simeq I\ \text{year}^{-1}$ (16). The influence of synchronization was not discerned within the limits of the materials on hand. Phase dispersion increases with time in accordance with the law of diffusion (21). The chaotic component of motion speed in the limit cycle is of the regular order (23).

The basic model of the central region of a quasar in which the cyclical changes in flux are considered are analyzed from the point of view of requirements imposed on the mechanism of variability. Preliminary considerations testify in favor of the oscillatory model, specifically, circulatory movements in a magnetoid[12-14].

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1. Introduction

Immediately following the remarkable discovery in 1963 of quasars by Schmidt[1] and Greenstein[2], systematic changes in the light flux from these sources were detected. Attempts to explain the variability were stimulated by the first considerations, namely that these were peculiar star-like objects in our galaxy in a critical phase of evolution. Evidence pointing to the extra-galactic localization of quasars[3,4] presented to our astrophysicists the remarkable fact that these objects of a non-stellar nature, sometimes vast cosmologic distances away, give off emanations which change comparatively quickly in time.

Mathews and Sandage[5] were the first to confirm this variability from the object 3C 48 on the basis of photoelectrical observation. Smith and Hoffleit[6], who studied a collection of photographs at the Harvard College Observatory gathered over a period of 80 years, established that there were similar variations in flux from 3C 273. A. S. Sharov and Yu. N. Yefremov[7] who studied the "glass library" plates at the Shternberg State Institute of Astronomy, as proposed by I. S. Shklovskiy, simultaneously and independently established the brightness variability in 3C 273. Sandage[8] pointed to the possibility of variability in the optical flux from 3C 196. In that case, the difference in brightness between two observations made seven months apart is considerably greater than any chance of a probable error. Not excluded is the probability of variability in other recently discovered quasars (3C 287, CTA-102 and others) in which successive observations of the degree of brightness have disclosed rather marked differences[9].

The peculiarities in flux variation from 3C 273[10] and 3C 48[8] are known in rather considerable detail today. A direct study (without any statistical processing of data) of the brightness curve of source 3C 273 enables us to

divide with confidence these changes into three components [6,10,11]: 1) very rapid chaotic changes (approximately by 60%) in the flux for periods of from one week to a month; 2) rather regular (cyclical) variations in brightness with changes of 30% in amplitude and over a characteristic period of 10 years; 3) a slow epochal flux approximately 30% per hundred years. The variable light flux from 3C 48 has similar properties. The three color photoelectrical observations made of this source[8] show changes in optical flux by 40% over a period of 600 days and 10% over several days; the epochal trend is a change of 30% per century[11].

It can be assumed that the property of variable radiation is an important property of quasars and is closely related to the processes occurring in their cores. Hence, a study of the property of variability can cast some light on the mechanism of physical phenomena in the cores of sources, which, for the time being, is the subject merely of theoretical syntheses. Moreover, in addition to dividing the variable flux into its components and developing a coarse upper limit of dimensions of the optical source[6] the information contained in the curve of brightness of the sources was not used anywhere. The study is the first attempt of its kind. In the following we shall give the results of a statistical analysis of cyclical variability in 3C 273, the most commonly studied object at the present time. We can expect that the property of regular variability inherent in this quasar source is typical of objects found in a similar stage of evolution. The necessary requirements for the mechanism of variability as obtained from an analysis of the brightness curve are given in Chapter III. These data can be compared with quasar models (Chapter IV) proposed at the present time. Along with an analysis[12,13,14] of certain general requirements for quasar models and the mechanism of their

variability this study points up some of ineffective aspects of certain models; on the other hand, these data provide the necessary information for a more thorough development of other models.

I. A Statistical Analysis of the Brightness Curve of 3C 273

The rather uniform materials we have on changes in brightness of 3C 273[10] enables us to study, for the time being, only the long period variations with a characteristic time of 10 years. This has to do with the fact that the data published[10] are an average of the original observations. Therefore, by way of preliminary treatment we slide smoothed the data off [skol'zyashcheye sglazhivaniye] with an interval of two years and eliminated them from the secular trend. The smoothing consisted of the following: for the year T_1 we extracted all the known values of brightness for the source in the interval $T_1 - 1 \leq t < T_1 + 1$ and took the mean from these taking into account the accuracy of averaged observations given in [10].

Selection of the smoothing interval value Δ was dictated, on the one hand, by the need to decrease the "external hum" (not pertaining to the signal shaped by the system though related to the conditions of propagation of the signal and various kinds of observation errors), and on the other hand, by a desire to preserve the greatest possible amount of detail in the information under analysis. Both these aims may be achieved if we select Δ as equal to 2 years. This smoothing process is, essentially, the filtration of a signal, which is frequently done in the statistical processing of materials. It should be noted that the filtration might have been improved if the original separate observations of brightness of 3C 273 had been available. The result of smoothing and reading off the trend are shown in Table I and in Fig. I. Here, and subsequently, the following scale is used for convenience: unity x = $0^m.1$.

The shape of Figure I confirms the conclusion regarding the existence of regular variations in flux with a duration of the order of 10 years. It would be purposeful to set up the following problem: approximate the curve x(t) by the simple vibration equation with one degree of freedom*):

$$\frac{d^2x}{dt^2} = f\left(x, \frac{dx}{dt}, \mu g(t)\right). \qquad (I)$$

If $\mu \equiv 0$, the system is autonomous. If $\mu \not\equiv 0$ and g(t) is the periodic function, we are dealing with a synchronized system. It is assumed that equation (I) has a stable periodic solution $x_0(t) \equiv x_0(t+T)$.

In a real vibrational system there are fluctuations as a result of which there are deviations in the signal [strength] from strict periodicity, so that the changes in signal [strength] in time assumes a cyclical character. Taking into account the small fluctuations, the equation changes in signal strength appears as follows:

$$\frac{d^2x}{dt^2} = f\left(x, \frac{dx}{dt}, \mu g(t)\right) + F(t). \qquad (Ia)$$

It is the existence of internal fluctuations in the system that makes it possible, on the basis of random deviations of a signal from a periodic solution, to find the characteristics of a fluctuating system without assuming any a priori judgements about it[15]. The most essential of the assumptions at which an analysis is fair[15], is to fulfill the conditions of a short correlation of fluctuation:

$$\langle F(t)\ F(t+T)\rangle \simeq 0 \text{ with } T > T_4 \ll T,$$

in which the sign $\langle\ \rangle$ is the mathematical expectation, T_4 is the interval of correlation, and T is the period.

* The use of an equation of a higher order seems premature, for the time being, due to the paucity of observation materials.

Setting up a concrete type of equation (Ia) on the basis of a study of the correlation properties of the signal being studied could be effected according to the method set forth in reference 16, but the low number of observed cycles permits us to offer only a linear approximation. In this approximation the limiting cycle (for an autonomous system) or the dynamic trajectory (for the captured system) coincide with the mean statistical trajectory describing the points of the system on the phase plane.

Having a sequence of points $x_i = x(t_l)$, $t_{l-1} - t_l \equiv 1$ year (cf. Fig. I and Table I) we find $y_i = \frac{1}{2}(x_{l+1} - x_{l-1})$, and successively connecting the points x_l, y_l of the smooth curve, we get a "stochatic trajectory" which describes the points of the system. In order to compute the mean statistical trajectory we use the method of successive approximations. We are looking for a closed curve with the following properties: any normal applied to it intersects it at a point which coincides (within the limits of errors imposed by the limited scope of selection) with the center of gravity of points of intersection of this normal with the stochatic trajectory[17].

The mean statistical trajectory obtained as a result of computations $x_0(\theta)$ is shown in Fig. 2. The period of the cycle proved to be T = 9 years, and the shape of the curve $x_0(\theta)$ can, within the limits of accuracy of the observations, be described as a sinusoid

$$x_0(\theta) = 1.25 \sin \frac{2\pi}{T}\theta \quad . \qquad (2)$$

It goes without saying that the amplitude of dynamic deviations (2) is smaller than the mean amplitude of changes in brightness of original data[10] in which both the internal fluctuation in the system and the "external noise" are superimposed on the dynamic pulsations.

Changing over to a local system of coordinates $\{n, \theta\}$, associated with the mean statistical trajectory, in a linear approximation we get the following equations[16]:

$$\frac{d\eta}{dt} + N\eta - \mu L_\theta \gamma = F_\eta \quad . \tag{3}$$

$$\frac{d\gamma}{dt} - \Theta\eta + K\gamma = F_\gamma \quad . \tag{4}$$

Here, η is the distance from the point described to the nearest point of the terminal cycle having the tangential coordinate θ; $\gamma = \theta - t$ is the tangential departure, F_η and F_γ are fluctuations which for an unsmoothed signal would be δ-correlated (with the natural assumption that the non-smoothed fluctuations are "white noise"), and following smoothing

$$\langle F(t)F(t+\tau)\rangle = \begin{cases} \frac{\langle F^2\rangle}{\Delta}(\Delta - \tau) & \text{where} \quad |\tau| \leq \Delta \\ 0 & \text{where} \quad |\tau| > \Delta \end{cases} \tag{5}$$

in which $t_1 \equiv \Delta = 2$ years is the interval of smoothing, and $\langle F^2 \rangle$ is the dispersion of smoothed fluctuations.

The meaning of the values N, θ, μ L, K is as follows. The coefficient N, hereafter referred to as rigidity [zhestokost'] characterizes, in the absence of fluctuation F, the speed of return of the point of description to the limiting cycle. The coefficient θ takes into account the influence of normal deviations from the limiting cycle on the speed of change in tangential departures γ of the imaging point and is related to N through the following relationship:

$$\Theta(\theta) = -\frac{\sin\varphi(\theta)}{y_0(\theta)}N(\theta) + \frac{\cos\varphi}{y_0(\theta)}\left(\frac{d\varphi}{d\theta} - 1\right), \tag{6}$$

in which $y_0(\theta) = dx_0(\theta)/d\theta$, and φ is the angle between the axis of ordinates and the normal to the limiting cycle. With $M \not\equiv 0$ (non-autonomous system) the coefficients L and K are determined by the shape of the synchronizing factor g(t) in (I) and they are related by the relationship:

$$K(\theta) = \mu L(\theta)\frac{\sin\varphi(\theta)}{y_0(\theta)} \quad . \tag{6a}$$

The short duration of recording x(t) (a total of 8 cycles) does not make it possible to obtain coefficients of equations (3) and (4) as position functions

on the terminal cycle. We shall have to restrict ourselves merely to an evaluation of the mean values of these coefficients by a method similar to that set forth in reference 16.

To evaluate the coefficients of the system (3) - (4) we computed functions $\overline{(\Delta\gamma_\tau)^2}$ and $\overline{(\Delta\eta_\tau)^2}$ which depicted the statistical nature of the deviations η and γ:

$$\overline{(\Delta\gamma_\tau)^2}=\frac{1}{k-\tau}\sum_{i=1}^{k-\tau}[\gamma(t_i+\tau)-\gamma(t_i)]^2 \quad , \tag{7}$$

$$\overline{(\Delta\eta_\tau)^2}=\frac{1}{k-\tau}\sum_{i=1}^{k-\tau}[\eta(t_i+\tau)-\eta(t_i)]^2 \quad , \tag{8}$$

in which τ changes from 1 to 30, while k = 71 is the number of observations shown in Table 1*.

In order to explain the relationship $\overline{(\Delta\gamma_\tau)^2}$ and $\overline{(\Delta\eta_\tau)^2}$ shown below in Figs. 3 and 4 with the coefficients of the system (3) and (4) we shall consider from the outset the more simple equation:

$$\frac{dz}{d\tau}+Az=F \quad , \quad z|_{\tau=0}=0 \quad . \tag{9}$$

Here, A = const., F is a random function with the following properties:

$$\langle F(t)F(t+\tau)\rangle=\Delta\cdot\langle F^2\rangle\delta(\tau) \; ; \; \langle F\rangle=0 \quad , \tag{10}$$

whereupon $\langle F^2\rangle$ is the dispersion smoothed off with an interval Δ; function F is a constant; $\delta(\tau)$ is a delta function.

Following a series of conversions we can get:

$$\langle(\Delta z_\tau)^2\rangle=\langle[z(t+\tau)-z(t)]^2\rangle=\frac{\Delta\cdot\langle F^2\rangle}{A}\left(1-e^{-A\tau}\right) \quad . \tag{11}$$

In boundary cases (11) gives us the following:

$$A\to 0 \; : \quad \langle(\Delta z_\tau)^2\rangle\to\Delta\cdot\langle F^2\rangle\cdot\tau \quad . \tag{12}$$

This case corresponds to the Vinerovskiy process described by the equation

$$A\to\infty \; : \quad \langle(\Delta z_\tau)^2\rangle\to\Delta\cdot\frac{\langle F^2\rangle}{A} \quad . \tag{13}$$

This situation corresponds to the random process of the same type as F and described by equation $z = F/A$.

* Taking into account the omission in Table 1 of data necessary for computation, the formulas are somewhat more complex.

If the function F in (9) will have, instead of the property (10), a terminal interval of correlation Δ similar to the right hand members in (3) - (4), i.e., $\langle F(t)F(t+\tau)\rangle \neq 0$ with $|\tau| > \Delta$, then the result (11) with will practically not change. In like manner, replacing the constant A in (9) by a periodically changing function will result in replacing in (11) the value A by its mean value for the period and the addition of oscillating members which will remain insignificant during the short period of recording.

We shall now refer to the original system (3) - (4) and compare equation (4) with equation (9). By way of a first approximation we shall digress for a time in (4) from member θn (in the following it will be demonstrated that disregarding it practically does not change the form $\langle(\Delta\gamma_c)^2\rangle$ even when $\tau \geq 2$). Then establishing the relationshp of value $\langle(\Delta\gamma_c)^2\rangle_1$, computed according to (7), with K and $\langle F_\delta^2$ from (4) is fully analogous to the derivation $\langle(\Delta\gamma_c)^2\rangle$ from (9) considered above. We shall consider the relationship of $\overline{(\Delta\gamma_c)^2}$ to τ in Fig. 3. The course of $\overline{(\Delta\gamma_c)^2}$ at large τ is close to linear, a fact that corresponds to case (12). In other words, in (4) K = 0, and on the strength of (6a) we will get $\mu = 0$.

The fact that coefficients M and K are equal to zero points to the system being autonomous, i.e., the absence in equation (1) of variable t. More specifically, it can be stated that no synchronization of systems is manifested, at least within the limits of the time interval - $3T \simeq 30$ years (it is impossible to determine, for the time being, whether synchronization exists at a greater interval since the time of observation would be about 80 years).

Equations (3) - (4) averaged for cycles with N = 0 and $K \simeq 0$ can be set down as

$$\frac{d^2\tilde{\gamma}_n}{dt^2} + \tilde{\gamma}_n = F_n \,, \tag{14}$$

$$\frac{d^2\tilde{\theta}_n}{dt^2} - \tilde{\theta}_n = F_\delta \,, \tag{15}$$

in which the symbol $\sim$ indicates averaging for cycle, after which $\tilde{N}$ = const, $\tilde{\vartheta}$ = const. We shall now determine coefficients $\tilde{N}$ and $\tilde{\vartheta}$. Equation (14) is analogous to equation (9) analyzed in the above, a fact which makes it possible, with the aid of (11) to get from the parameters of the curve the approximated course of $\overline{(\Delta h_\tau)^2}$ (Fig. 4), and the average rigidity $\tilde{N}$ and $\langle F_h^2 \rangle$. By the method of the least squares we get from Fig. 4

$$\tilde{N} \simeq 1 \ [\text{год}^{-1}] \ , \tag{16}$$

$$\langle F_n^4 \rangle \simeq \frac{0.6\,\tilde{N}}{\Delta} \simeq 0.3 \ [\text{год}^{-2}] \ . \tag{17}$$

It is difficult to indicate the exact value of $\tilde{\epsilon}$ since the relationship between $\tilde{\epsilon}$ and $\tilde{N}$ is obtained by averaging (6) which depends, to a marked degree, on the specific form of $N(\theta)$; nevertheless, taking into account (2) we can consider

$$|\tilde{\Theta}| \leq \tilde{N} \cdot 1\,\text{год} \ . \tag{18}$$

We shall now demonstrate that to evaluate K and M in the system (3) - (4) we were justified in disregarding member ϵn. From (14) - (15) we can get the following expression for $\langle (\Delta\gamma_\tau)^2 \rangle$

$$\langle (\Delta\gamma_\tau)^2 \rangle = \Delta \cdot \left\{ \langle F_g^4 \rangle + \frac{\tilde{\Theta}^2}{\tilde{N}^2} \langle F_n^2 \rangle \right\} \tau - \frac{\tilde{\Theta}^2}{\tilde{N}^3} \cdot \Delta \cdot \langle F_n^2 \rangle \left(1 - e^{-\tilde{N}\tau} \right) . \tag{19}$$

With $\tau \gtrsim 2$ from (19) taking into account (16) it follows

$$\langle (\Delta\gamma_\tau)^2 \rangle = a\tau - b \ ;$$
$$a \equiv \Delta \cdot \left(\langle F_g^2 \rangle + \frac{\tilde{\Theta}^2}{\tilde{N}^2} \langle F_n^2 \rangle \right) \ ; \ b \equiv \frac{\tilde{\Theta}^2}{\tilde{N}^3} \cdot \Delta \cdot \langle F_n^2 \rangle . \tag{20}$$

From (20) and (17), (18) it follows that $b \lesssim 0.6$; as we can see from Fig. 3 the influence of this member, bound to member δn in equation (4) with $\tau \gtrsim 2$ is negligibly small; this demonstrates the validity of the inference made.

Thus, the relationship of $\overline{(\Delta\gamma_\tau)^2}$ to τ represented in Fig. 3 can be approximated as

$$\overline{(\Delta\gamma_\tau)^2} = a\tau \tag{21}$$

with coefficient 2 determined from Fig. 3

$$a = 1{,}4 \div 1{,}8 \quad *) \tag{21a}$$

Taking into account the fact that $0 \leq \tilde{\Theta}^2 \langle F_n^2 \rangle / \tilde{N}^2 \leq 0.3$ we will compute from (20) and (21)

$$\langle F_\gamma^2 \rangle = \frac{a}{\Delta} - \frac{\tilde{\Theta}^2}{\tilde{N}^2} \langle F_n^2 \rangle \simeq 0.4 \div 0.9 \tag{22}$$

On the other hand, it can be demonstrated that $\langle F_\gamma^2 \rangle \simeq \langle F_n^2 \rangle \left(\frac{T}{2\pi}\right)^2$, so that with $T \simeq 9$ years

$$\langle F_\gamma^2 \rangle \simeq 0.6 \,. \tag{23}$$

The good agreement of estimates (22) and (23) made by various methods points to the internal agreement of approximations used.

Finally we can assume that $\langle F_\gamma^2 \rangle \simeq 0{,}6$. Having only an estimate of average rigidity (and not rigidity in each point of the limiting cycle) and a situation in which the deviation from the limiting cycle caused by internal fluctuations in the system are commensurable with deviations due to "external noise" (errors in observation, influence of the medium in the propagation of the signal, etc.), it is not desirable to use equation (14) to describe and, in particular, to forecast the signal x(t). The maximum of proved information which could be extracted from an analysis of normal deviations is the evaluation of the mean rigidity N and the value $\langle F_n^2 \rangle$. In contrast to normal drifts from the limiting cycle the diffuse phases [razmytyye fazy] increase in time due to tangential drifts and very soon pass beyond the limits of observation error. This circumstance should be taken into account even in

* There are several reasons for the slight deviation from linearity which occurs in Fig. 3: paucity of statistics, linearization of equations with exchange of coefficients changing with phase θ to constant values [e.g., if $N(\theta)$ is of a pulsing character as is the case with the sun [17] the deviation from linearity should be observed to $t \simeq T$] etc. However, as we can see from Fig. 3 the deviations are not very great, and in conformance with the previously accepted accuracy of analysis the linear approximation is fully admissible.

the coarsest approximation in describing a signal. In conformance with the conclusion about the autonomous state of the system (K = 0) we can approximate the signal in the following manner

$$x(t) \simeq x_0(\theta(t)) \equiv x_0\left(t + \gamma(t)\right) \; ; \quad \gamma = \int F_\delta \, dt \; ;$$
$$m = 0.1\, x = 0.125 \sin \frac{2\pi}{T} \int_0^t (F_\gamma + 1)\, dt \; ; \quad T = 9_{\text{лет}}, \; \langle F_\gamma^2 \rangle \simeq 0.6 \; . \tag{24}$$

We will compute the accuracy of the value obtained for a period of $T \simeq 9$ years. This accuracy is determined by the length of the time series of observation. Designating the moment of advent of a fixed phase θ for m - 20 cycle through t_m we have the following obvious equalities:

$$t_{m+1} - t_1 = mT + \gamma(t_1) - \gamma(t_{m+1}) \; ;$$
$$T = \frac{t_{m+1} - t_1}{m} + \delta T \; ; \quad \langle \delta T \rangle = 0 \; .$$

For the dispersion of random value δT we get

$$\langle (\delta T)^2 \rangle = \frac{\langle [\gamma(t_{m+1}) - \gamma(t_1)]^2 \rangle}{m^2} \; ,$$

or, using the previously determined values $\langle F^2 \rangle$ and Δ ,

$$\langle (\delta T)^2 \rangle = \frac{\Delta \langle F_\gamma^2 \rangle T}{m} \; ,$$

which was m = 8, $\langle F_\gamma^2 \rangle = 0.6$, T = 9, and $\Delta = 2$ gives $\langle (\delta T)^2 \rangle = 1.35$.

Thus, the standard error in determining the period

$$\sqrt{\langle (\delta T)^2 \rangle} = 1,2 \text{ years.} \tag{25}$$

Individual cycles have standard deviations that are $\sqrt{m}$ greater than (23) which with a mean duration of the cycle 9 ± 1.2 years is equal to $\simeq 3.4$ years. Obviously this does not contradict the observed duration of the two last cycles on the brightness curve of 3C 273 by about 13 years. We will mention, in passing, that Schmidt[10] mentioned the value of T = 13 years as the period of change in flux from 3C 273; apparently he bore in mind only the last two cycles. The use of this value as the characteristic of the vibrational system period is not convincing in view of the above indicated changes in duration of individual cycles (we should add that the first cycles

observed had a duration of 7-8 years, as can be seen from Fig. 1); we are more justified in using a mean value found by a method like the one set forth in the above. It should also be mentioned that the materials at hand do not allow us to judge as to whether systematic changes exist in the continuance of the cycle with the passage of time.

III. Results of analysis

On the basis of the foregoing we have reached the following conclusions:

1. The period of dynamic movement of the system studied is $T = 9 \pm 1.2$ years.

2. The system is non-linear, the mean rigidity is positive, and $\tilde{N} \simeq 1$ year^{-1}. The existence of rigidity insures stability to the system, i.e., it returns to move in the limiting cycle with slight changes in initial conditions.

3. On the basis of the materials on hand synchronization is not manifested so that we can speak about the autonomous nature of the system. A consequence of this is the diffusional law of increase in phase dispersion with the passage of time[21].

4. The periodic solution corresponding to the movement according to the limiting cycle is $m = 0^m,125 \sin \frac{2\pi}{T} \theta$.

5. The chaotic component of velocity of movement in the limiting cycle is of the regular order ($\sqrt{\langle F_n^2 \rangle} \sim 1$).

We will make several remarks pertaining to the reliability of the conclusions made. The smallness of the time series (8 cycles in all) imposes a specific impression on the statistical analysis made. We are seeking only mean estimates for four parameters: $\tilde{N}$, $\langle F_n^2 \rangle$ (according to curve in Fig. 4) and a, K (on curve in Fig. 3). Estimates of all the other values are derivatives of these basic values. In addition, it seems that one of the

parameters is very small ($K \simeq 0$). Thus, from the curve consisting of 70 points we estimate three parameters, which is quite usual in statistical practices.

It is difficult to state whether the mathematical model (essentially an approximational one) will be retained within the previous bounds or whether it will become more complex in measure as the time series of observations is extended; but one way or the other it is certain that the values of the parameters determined above will not change very greatly.

IV. Comparison with theoretical variability models

A short time after the discovery of variability in radiations given off by quasars a variety of ideas was expressed as to the reasons for this phenomenon. It is hard to disagree with the concept that variability in radiations by quasars is such a fundamental property that its mechanism should, in the final analysis, be explained within the framework of concepts dealing with the very nature of the sources*. More specificially, the mechanism of variability should be tied in with processes occurring in the central region of the quasar whence the luminous continuum emanates. The question as to whether the quasar is evolved from a sequence of radio galaxies as a special, morphological type that had no precedent, or whether it is a special stage of evolution of already known radio galaxies should be resolved by other methods. In particular, there are definite bases[19,20] to assume the quasi-stellar objects are related to the Safertovskiy galaxies. In any case, the central region of the quasar and its core, which yields a considerable portion if not

* To be sure there was the opinion expressed by Greenstein and Young[10], that variability may be tied in with the changing transparency of the hydrogen cloud which surrounds the central source. Making the point more concrete, Schmidt assumes that the luminous variations occur when light intersects the NP region about the quasar if this region has a filamentary structure. No evaluations tending to demonstrate the possibilities of this mechanism were cited in [18]. From what is set forth in the following it appears that the situation is improbable.

the entire uninterrupted spectrum changing with time, presents the greatest riddle and develops the quasar as a specific phenonmenon. Not surprising, in this connection, is the fact that attempts to resolve the problem of quasars are based, mainly, on the hypothesis regarding the nature of their nuclei. Hence, models currently extant of quasars will be analyzed only from the point of view of the extent to which the mechanism of variability tied in with this model satisifes the necessary observation requirements obtained in Chapter II and summarized in Chapter III. In this, it is necessary to exclude from consideration those of the existing quasar models which do not contain a non-constrained mechanism of variability or which merely make a pretense at explaining the short durability, sporadic changes in brightness.

It should be mentioned that many models of quasars as well as mechanisms of variability are based on concepts of the thermal nature of radiation. Careful observations recently made have demonstrated the inconsistency of this view[5,21,22]; radiation from the core of a quasar [in the case of 3C 273 it is mainly infrared] is magneto-decelerating 5, 21, 23, 24. Hence, certain models in the variation proposed are no longer needed and should be modified essentially, if possible.

Models of quasars in which long periodic changes in luminous emittance are studied can be divided into three categories insofar as the mechanisms of variability are concerned: a) imposition of random events; b) rotatory models; c) oscillatory models. We shall consider each one individually.

a) Imposition of random events

Hoyle and Fowler[25] regarded the quasar as a small region which had undergone fragmentation into a multiplicity of objects. Because of the proximity to one another, the objects are subject to frequent collision with the

resultant great liberation of energy. Reference 25, in speaking of sources of variability, points to the rapid nuclear evolution of individual fragments. A similar idea that preceded 25 was discussed by Walter[26]; in a model of this idea the luminous variations were due to collisions of neighboring fragments. The most highly developed model of this type was proposed by Field[27]. According to 27 the quasar is a spheroidal galaxy in the stage of intensive star formation. Variations in luminous flux are related to flares of the supernova II type. To insure an average luminosity of 4.10^{45} erg/sec it is necessary to have about 50 supernovas; it is confirmed that by imposing independent explosions we can get a rough approximation of the observed picture of variability. Understanding the contradiction between the short duration of illumination of a supernova compared with the required time scale of variability, Filts turns to the mechanism proposed by Colgate and Cameron (to explain the flare phenomena in quasars)[28], according to which radiation occurs following the interaction of the shock wave, created by the supernova, upon the surrounding very dense gas.

The concepts developed by Field run counter to a number of serious objections (cf also 30). Since the energetic maximum of radiation of 3C 273 lies in the infrared and submillimeter region[22], so that the yield in the uninterrupted spectrum constitutes 3.10^{47} erg/sec[24] (cf also 23), it is necessary to involve in the process of star formation a cloud with a mass of up to 10^{12} M , which is a great deal larger than the ordinary mass of an elliptical galaxy. Further, the clearly expressed and stable periodicity of the smoothed curve of brightness (Fig. 1) cannot be reconciled with the random processes of supernova flares considered in reference 27. In addition, the requirement for a number of supernovas, in connection with the new observations[22] is raised by an order compared with 27, and this results in a still

greater "erosion" of the overall brightness curve. The nature of the spectrums of quasar objects reminds one very little of the spectrums of supernovas. Thus, the peculiarities of changing radiations of a quasar cannot be explained within the framework of a Field model if we do not turn to a hypothetical ensemble of objects related to one another in a definite manner, which are different from conventional supernovas.

A more general conclusion is that all mechanisms of variability (cf also 14) which are based on the imposition of a large number of random events uniformly distributed in time* should have a very broad and smooth spectrum of frequencies whereas the variability of 3C 273 shows a well expressed nine-year harmonic.

b) Rotatory models

The suggestion was made in reference 25 that long period variations can also be caused by the rotation of the nucleus. A similar variation within the framework of the magneto-decelerating nature of radiation was considered in 13 in which is discussed the relationship of this rotation to the movement in the emission shell of 3C 273.

The proposal is made in reference 13 that the rotating nucleus is supported in the quasi-stationary condition by magneto-turbulent pressure. Rotation is one of the alternate possibilities involving period modulations of the magnetic field in which there is a radiation of relativistic electrons. Variability can take place in accordance with reference 13 if the line of sight intersects a quasi-regular heterogeneous field given a rotating region with variable intensity. It is assumed here, that the cycle of change of brightness coincides, on the average, with the period of rotation. Sporadic

* The heterogeneous distribution of random events in time actually points to a synchronizing mechanism, i.e., it makes the model more complex without resolving any of the difficulties.

perturbations caused by turbulence can be imposed on the regular variability.

Analysis of this model leads us to conclude that the imposition of "noise" on a periodic solution produced by rotation will result in a constant dispersion of phase in the outgoing signal. Moreover, according to the analysis made above, (cf. pt. 3 in Chap. III), the phase dispersion grows monotonically with time. In addition, the average rigidity of the rotating system, given a symmetrical form of limiting cycle, is equal to zero, whereas the observed mean rigidity is $\tilde{N} \sim 1$. Both these circumstances enable one to conclude that rotation with a period close to the observed duration of cycle is not the direct reason for the variability.

An alternate possibility is the case in which the rotation of source is considerably slower than the duration of the cycle. Here, the variability is caused by the intersection of the line of sight by heterogeneous particles of the surface of the quasar nucleus. In other words, the variability observed is a temporary development in the spatial plane of these sectors - "lobes". By way of example, the eight cycles observed from this point of view could be interpreted as the result of the passage of eight "lobes" in the plane of the sky. In this connection, the conclusion 3, Chapter III (the growth in phase dispersion with time) can be satisfied. However, the large value for rigidity obtained in the above contradicts this modification of a rotatory model. Actually, to explain the required rigidity, it would be necessary for the deflection of radiation of each "lobe" from the mean to be not random, but selective in a very special manner. Similar, very artificial requirements of the "lobes" should be set forth if we consider such a multi-lobed "air vane" as an absorbing system and not a radiating system (cf. remarks on [Russian] page 15). [translation page 14]

c) Oscillatory models of variability

We include in this group models in which variability is insured by different oscillatory rates.

First we shall consider attempts to interpret variability of flux as the result of pulsations of a massive object[10,29]. In [10] the parameters of a pulsating massive star were evaluated from the well known relationship $T(\bar{\rho})^{1/2} \simeq \varphi^{-1/2}$ involving the period of pulsation of the gas sphere with its mean density; for the nucleus of quasar 3C 273 we got $\hat{\rho} \simeq 2.10^{-10}$ g/cm^3, $A \simeq 2.10^{-16}$ cm and $M = 3.10^6 M_\odot$. However, the relationship used is not suitable for massive stars whose equilibrium is not determined by the gas pressure but by the beam pressure, and the correct relationship will contain a factor of the order $(M/M_\odot)^{1/4}\ (S_c/\bar{S})^{-1/2}$ on the right-hand side[29]. More important is the fact that it is impossible to insure, by the thermal radiation of such a "superstar", the necessary power of radiation for any reasonable time. By way of example, the necessary output of $\sim 10^{47}$ ergs/sec requires $M \quad 10^9 M$; the balanced existence of the superstar of such a mass is impossible[35,36]. In references 12 and 13 the proposal was made that the quasar nucleus is maintained in a quasi-balanced state by a magnetic field. The magneto-decelerating radiation in a changing field is regarded as a mechanism of variable brightness of objects. Alternate possibilities of modulation of a magnetic field, together with the rotation considered in the foregoing, are the pulsations of a magnetic superstar and the circulating movements of plasma in the nucleus.

In pulsations of a superstar the lines of force in its magnetic field that tie the surface of the star with the magneto-sphere of the nucleus (from which the observed flux radiates) are subject to oscillations which modulate the magnetic field in the magnetosphere. As a specific example, reference 13 discusses a magnetic superstar with a period of pulsation of 10 years with

$M = 10^5 M_\odot$, $R = 2.10^{15}$ cm, whose magnetic field at a distance of 10^{16} cm is $\sim 10^2 - 10^3$ oersteds, and this is necessary to explain the optical and infrared continuum in 3C 273[23]. The numerical values given are illustrative, but it would be interesting to know how they can be reconciled with observation requirements. This helps to explain the epochal changes in the brightness of quasar[13].

The pulsation in the superstar is maintained due to convection which occurs in the strong magentic field of the superstar as a variable instability[12, 13]. The mechanism of fluctuation of a superstar is not computed in detail. Along with this, if the oscillations of a magnetic superstar are really responsible for changes in brightness of a quasar the conclusions made in Chapter III are undoubtedly important as requirements for the mechanism of pulsation. Very important is the conclusion made about the non-linearily of the process of variation; it is interesting to compare it with the process of establishing the amplitude of variation in a convection impeded by a magnetic field. Likewise, the growth of phase dispersion in this fluctuating system appears quite probable.

Another variation in the pulsating superstar was recently proposed by Fowler[31]. In his model the superstar is constricted from $R \sim 10^{17}$ cm to $R \sim 10^{13}$ cm, following which it returns to the original state due to the generation of energy by the nucleus. These ideas[14] run into difficulties such as: the ["perekhlest"] "lashing" is capable of destroying a star, the rapid fluorescence in the stage of maximum expansion is not taken into account, and others. Special emphasis is put on the fact that we can hardly identify the entire quasar nucleus with the superstar as proposed in references 31 and 10.

In this connection, it is only natural to consider alternate possibilities for modulating the field in the nucleus, but without reference to the superstar.

This possible mechanism is insured by the very nature of maintaining the nucleus in equilibrium by the magnetic field[12,13]. Specifically, the vortical character of the magnetic field prevents getting static equilibrium, but it does not exclude a quasi-stationary condition. The evolution of a quasar nucleus consists in going through a series of balanced configurations with a developed magnetic turbulence[32,33]. Such a configuration, consisting of a formation with powerful magnetic fields and strong movements which balance the force of gravitation, is called a "magnetoid"[33]. The evolution of a magnetoid may be associated with the evolution of a constricting massive cloud in which the magnetic energy was originally comparable to the gravitational energy and upon constriction it grew more rapidly due to the flow of rotational energy and the energy of chaotic movements into magnetic energy. The large scale circulation of heterogeneities with the "frozen in" magnetic field could explain the cyclical changes in flux from the quasar nucleus. The turbulence superimposed on the main movement can be related to the fluctuation and sporadic "flare" type phenomena.

One of the more simple models of a magnetoid, which is a precise solution of a magnetic gas dynamics equation, is the rotation of the plasma along the lines of force of a toroidal magnetic field. The period of rotation with a nuclear mass of $10^8 M$ and a characteristic dimension of 4.10^{16} cm is of the order of several years[12] -- and that is what is required. This model gives us the correct order of magnitude for the time of circulation of heterogeneity of a magnetic field in more complex models for which a precise solution and the study of its stability is fraught with numerous difficulties.

Given a circulating movement of the plasma in the quasar nucleus the presence of turbulence should bring about a condition in which the preceding cycle imposes a weak impression on the succeeding one: one turn will occur

more rapidly due to fluctuations, and the one succeeding will hardly "notice" it. Therefore, phase dispersion will increase, just as required by observation (cf. Chapter III). The presence of a large scale movement insures rigidity to a given oscillatory system, and this can also be reconciled with the necessary observational requirements. With a rigidity of $\tilde{N} \simeq$ I year^{-1} the time for suppressing the fluctuation $t \sim$ I year, as we can see from (14). The dissipation of energy of corresponding fluctuating movements can be insured by the generation of cosmic rays.

Hence, among the models considered, the oscillatory model is the most probable one to explain the mechanism of variability in a quasi-stellar object. Although a numerical evaluation of the relative role of a magnetic superstar and circulating movements in producing variability were not made there is basis to regard the second as the more probable reason for the variability. Such a selection should be made if the appearance of the phenomena of quasi-stellar sources is recurrent in character[19,34] and variability is present in all phases between successive explosions. Actually, the time of life of a superstar with a mass of $10^5 M_\odot$ is of the order of 10^4, whereas[36] the intervals between flares are of one to two orders greater if we are to judge, for example, by the distance of the ejections.

Here, we did not concern ourselves with an analysis of the possible models of circulating movements and comparing them with the inferences in Chapter III. These problems are the subject of further investigations.

The authors are grateful to L. I. Gudzenko for his guidance in this article.

Table I

	x	y	Θ	γ	n		x	y	Θ	γ	n
I888	0,7	-0,9	6,0	0	0,I	I928	-I,2	0,7	8,5	7,5	0,2
I889	-0,2	-I,0	7,0	0	0,2	I929	0,2	I,2	2,5	9,5	0,2
I890	-I,2	-I,0	8,0	0	0,4	I930	I,2	0,9	3,5	9,5	0,3
I89I	-2,2	-0,4	8,5	-0,5	I,4	I93I	2,0	0,3	4,0	9,0	0,7
I892	-2,0	0,I	0,0	-I,0	0,7	I932	I,7	-0,2	5,0	9,0	0,2
I893	-2,0	0,4	0,5	-I,5	0,8	I933	I,7	-0,5	5,0	8,0	0,5
I894	-I,I	I,I	I,0	-2,0	0,6	I934	0,7	-0,8	6,0	8,0	0,0
I895	0,2	I,0	2,5	-I,5	0,0	I935	0,0	-0,6	7,0	8,0	-0,3
I896	0,9	0,6	3,5	-I,5	-0,I	I936	-0,6	-0,6	7,5	7,5	-0,2
I897	I,4	0,I	4,5	-I,5	0,I	I937	-I,2	0,3	0,5	8,5	0,0
I898	I,I	-0,7	5,5	-I,5	0,I	I938	-I,2	0,0	0	7,0	-0,I
I899	0,0	-0,6	7,0	-I,0	-0,3	I939	-I,3	-0,2	9,0	6,0	0,I
I900	0,0	-0,4	7,0	-2,0	-0,5	I940	-I,7	-0,4	8,5	4,5	0,4
I90I	-0,7	-0,6	8,0	-2,0	-0,2	I94I	-2,I	0,I	9,0	4,0	0,8
I902	-I,3	-0,I	9,0	-2,0	0,I	I942	-I,4	0,6	I,0	4,0	0,4
I903	-0,9	I,I	I,5	-I,5	0,4	I943	-0,9	0,6	I,0	3,0	0,0
I904	I,0	I,4	3,0	-I,0	0,6	I944	-0,3	0,8	2,0	3,0	-0,I
I905	I,9	0,I	4,5	-0,5	0,6	I945	0,6	I,0	3,0	3,0	0,3
I906	I,2	-0,6	5,5	-0,5	0,2	I946	I,7	0,I	4,5	3,5	0,4
I907	0,6	-0,6	6,0	-I,0	-0,2	I947	0,8	-0,6	6,0	4,0	-0,2
I908	0,0	-0,7	7,0	-I,0	-0,2	I948	0,5	-0,7	6,5	3,5	-0,2
I909	-0,9	-0,2	8,5	-0,5	-0,4	I949	-0,6	-0,7	7,5	3,5	-0,I
I9I0	-0,5	0,6	I,5	0,5	-0,2	I950	-0,3	0,0	9,0	4,0	-0,4
I9II	0,4	0,7	2,5	0,5	-0,2	I95I	-0,6	0,0	9,0	3,0	-0,6
I9I2	0,8	0,3	3,5	0,5	-0,3	I952	-0,7	-0,2	8,5	I,5	-0,5
I9I3	I,0	0,I	4,0	0,0	-0,3	I953	-I,0	-0,2	8,5	0,5	-0,3
I9I4	I,0	-I,2	6,0	I,0	0,5	I954	-I,I	0,2	0,5	0,5	-0,I
I9I5	-I,4	-0,8	8,5	2,5	0,4	I955	-0,6	0,3	I,0	0,0	-0,4
I9I6	-0,6	0,4	I,0	3,0	-0,3	I956	-0,4	0,2	I,5	-0,5	-0,6
I9I7	-0,5	0,I	I,0	2,0	-0,6	I957	-0,2	I,I	2,0	-I,0	0,2
I9I8[1]	Данные отсутствуют или неуверенные					I958	I,8	0,6	4,0	0	0,7
						I959	I,I	-0,8	5,5	0,5	0,2
I923	-2,I	0,8	0,5	4,5	0,9	I960	0,2	-0,8	6,5	0,5	-0,I
I924	0,3	I,4	2,5	5,5	0,4	I96I	-0,3	-I,I	7,5	0,5	0,I
I925	0,7	0,0	4,5	6,5	-0,6	I962					
I926	0,4	-0,9	6,5	7,5	0,0		-2,3	-I,0	8,3	0,5	I,0
I927	-I,2	-0,8	8,0	8,0	0,3	I963	(-2,4)	(-0,4)	(9,5)	(9)	(1,2)

1) Data lacking or not confirmed

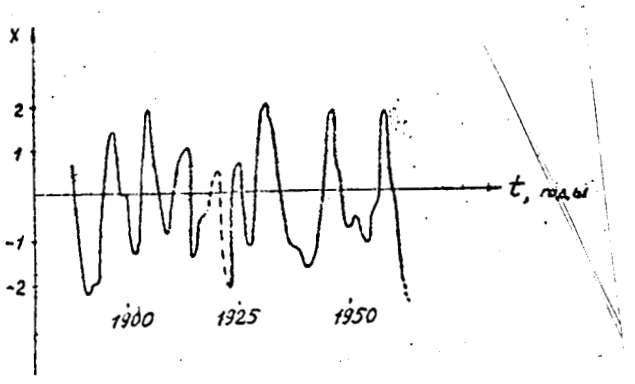

Fig. 1. Brightness curve of 3C 273 following slide smoothing and after being cleared of secular trend.

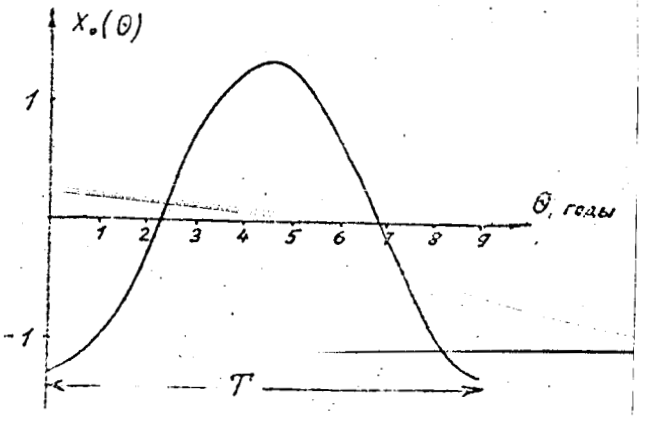

Fig. 2. Mean statistical trajectory.

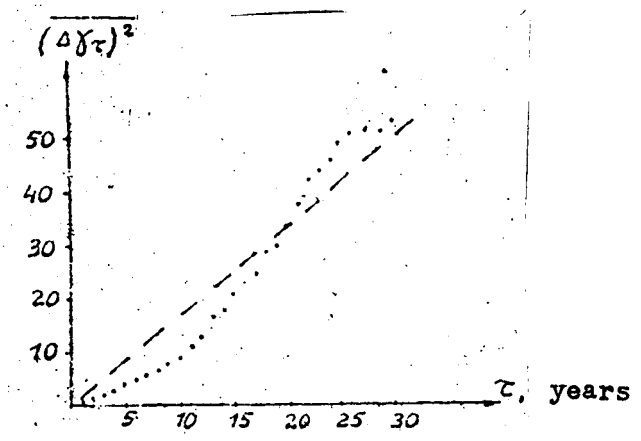
$(\Delta\gamma_\tau)^2$
50
40
30
20
10
5
10
15
20
25
30
τ, years

Fig. 3.

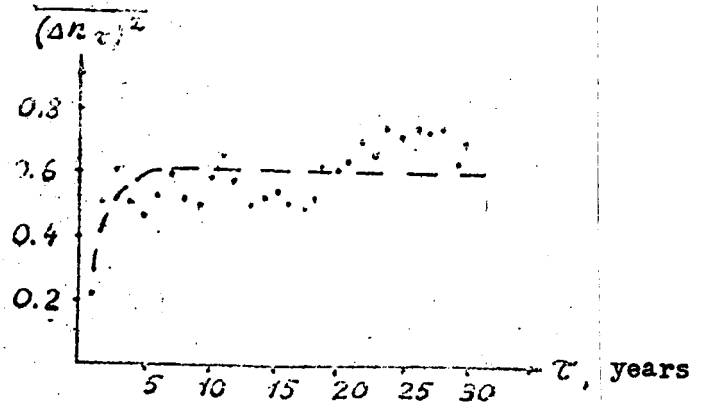
$(\Delta n_\tau)^2$
0.8
0.6
0.4
0.2
5
10
15
20
25
30
τ, years

Fig. 4.

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